HYDROMAGNETIC STABILITY OF A STREAMING COMPRESSIBLE GAS CYLINDER IN A LIQUID

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Abstract

The hydromagnetic instability of compressible hollow jet endowed with surface tension is discussed in the axisymmetric mode for all short and long wavelengths.
The dispersion relation is derived and discussed analytically and numerically.

Keywords: Self-gravitating, Electrodynamic, Streaming, Stability

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1- Introduction

The main prerogative of the present paper is to investigate the hydromagnetic stability of a streaming compressible hollow cylinder endowed with surface tension and acting upon its inertia and the electromagnetic forces. The stability and oscillation of full liquid jet endowed with surface tension or/and acted by electromagnetic force have been documented in several reported works based on the linear perturbation technique of small disturbance. See Rayleigh (1945) ,Lin (1976), Drazin and Reid (1980), Chandrasekhar (1981), Avital (1995) and Radwan (2004). The instability of hollow jet (gas cylinder penetrated in a liquid) acted by surface tension only is envised and studied for first time in the scientific province by Chandrasekhar (for axisymmetric mode (m=0) , m is the azimuthally wave number)only . Also Drazin and Reid (1980) and Kendall (1986) gave an idea about such problem to be done mathematically for axisymmetric and non-axisymmetric.In such work Channdrasekhar (1981), the inertia of the liquid is considered to be predominate over that of the gas and consequently the gas inertia force is neglected. Cheng (1985) elaborated the capillary stability of a streaming gas jet in a liquid, taking into account that the inertia of both incompressible gas and liquid. However one has to infer here that the result given longitudinal wavenumber and R_0 is the cylinder radius in the equilibrium state) must be in the numerator as it is clear from Eq.(3) in Cheng (19 by Cheng (1985), in Eqs. (4) and (5), are incorrect in the third term. In fact the term $\left(1-m^2-k^2R_o^2\right)$,(where m is the azimuthally wavenumber, k is the l85) . See also equations (2.45),(2.46) and (2.48) in the present work and Drazin & Reid's result (1980) p.16 and also Chandrasekhar's dispersion relation (1981) p.538 and p.540 (Eqs. (147) and (155) there). Radwan (1991) has examined the effect of a magnetic field on the capillary instability of an incompressible inviscid hollow jet. The stability of different cylindrical models under the action of self gravitating force in addition to other forces has been elaborated by Radwan and Hasan (2008) and (2009). They (2008) studied the gravitational stability of a fluid cylinder under transverse time-dependent electric field for axisymmetric perturbations. Hasan (2011) has discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, self gravitating and electrodynamic forces for all axisymmetric and non axisymmetric perturbation modes. He (2012) studied the magnetodynamic stability of a fluid jet pervaded by transverse varying magnetic field while its surrounding tenuous medium is penetrated by uniform magnetic field.

Here we extend the latter works by considering the liquid is compressible, which means that the velocity is not solenoid anymore and that the density is not constant.

2- Formulation of the Problem

We consider a hollow cylinder which is a gas cylinder pervaded into a liquid. In the initial state the gas cylinder is of cross section of radius R_0 . The liquid is assumed to be non-viscous, perfectly conducting and compressible (i.e. its density ρ will not be constant) and pervaded by the uniform magnetic field $\underline{H}_0 = (0,0,H_0)$. The gas is pervaded by the uniform magnetic field $\underline{H}_0^g = (0,0,\alpha H_0)$ where H_0 is the intensity of the magnetic field in the unperturbed state, while α is parameter satisfying certain restrictions. The components of the vector fields $\underline{H}_0$ and $\underline{H}_0^g$ are considered along the cylindrical coordinates (r,φ,z) system with the z-axis coinciding with the axis of the hollow cylinder model. Each of the gas and liquid is considered with constant magnetic permeability.

3 - Basic Equations

The basic equations concerning MHD study of compressible fluid

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\nabla)\underline{u}\right) = -\nabla P + \mu(\nabla\wedge\underline{H})\wedge\underline{H} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{u}) = 0 \tag{2}$$

$$\rho C_v\left(\frac{\partial T}{\partial t} + (\underline{u}\cdot\nabla)T\right) = -P(\nabla\cdot\underline{u}) \tag{3}$$

$$P = K\rho^{\gamma} \tag{4}$$

$$\nabla.\underline{H} = 0 \tag{5}$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla\wedge(\underline{u}\wedge\underline{H}) = (\underline{H}\cdot\nabla)\underline{u} - \underline{H}(\nabla\cdot\underline{u}) - (\underline{u}\cdot\nabla)\underline{H} \tag{6}$$

In the gas region

$$\nabla\cdot\underline{H}^{gas} = 0 \tag{7}$$

$$\nabla\wedge\underline{H}^{gas} = 0 \tag{8}$$

Along the gas-liquid interface, the surface pressure due to the capillary force is given by

$$P_s = S(\nabla.\underline{N}) \tag{9}$$

with

$$\nabla.\underline{N} = r_1^{-1} + r_2^{-1} \tag{10}$$

Here $\underline{u}$ and P are the liquid velocity and kinetic pressure, $\underline{H}$ is the magnetic field intensity, T is the temperature of the liquid, C_v is the specific heat of constant volume, $\gamma(=(C_p/C_v))$ is the ratio of specific heats of the liquid, S is the surface tension coefficient, while r_1 and r_2 are the principle radii of curvature. $\underline{N}$ is, a unit vector outward normal to the performed interface f(r,0, z, t) = 0, given by

$$\underline{N} = \nabla f(r,0,z,t) / \left|\nabla f(r,0,z,t)\right| \tag{11}$$

Equation (1) is the equation of motion of the liquid, equation (2) is the continuity equation of the liquid in the case of the compressible fluid, equation (3) is the conservation of energy equation, equation (4) is the polytrophic equation of state valid only for compressible fluids, equation (5) is Gauss's law of the magnetic field and this equation is identically satisfied, equation (6) is the evaluation equation of the magnetic field in the liquid region, equation (7) is Gauss's law of the magnetic field in the gas region, and equation (8) is the equation of conservation of flux in gas region where there is no current.

4 - Unperturbed State

In the unperturbed state, we consider the liquid streams with the velocity

$\underline{u}_0$ = (0, 0, U). The unperturbed state is studied and consequently the kinetic pressure of the liquid is given by

$$P_0 = -\frac{S}{R_0^2} + \frac{\mu H_0^2}{2}\left(\alpha^2 - 1\right) + P_0^{gas} \tag{12}$$

In the absence of the capillary force effect (S = 0), the pressure P_0 is positive as long as ($\alpha > 1$). However the model will collapse as ($\alpha = 1$) if $P_0^g > \left(S / \; R_0^2\right)$. Also if we neglect the surface tension effect, so P_0^g must be greater than $\left(S / \; R_0^2\right)$ to avoid the collapsing of the model.

5 - Linearization

We assume a small disturbance along the gas-liquid interface, then for small departure from the unperturbed state, every physical quantity $\chi(r,\varphi, z, t)$ may be expressed, see Radwan (2004) and (1996) as

$$\chi(r,\varphi,z,t) = \chi_0(r) + \varepsilon(t)\chi(r,0,z) \tag{13}$$

where the subscript zero characterizes quantities in the initial state while those with the index unity are their increments. Here χ stands for ρ, P, $\underline{u}$, $\underline{H}$, $\underline{H}^{gas}$, $\underline{N}$ and the radical distance of the gas cylinder. The amplitude of the perturbation $\varepsilon(t)$ is given by

$$\varepsilon(t) = \varepsilon_0 \exp(\sigma t) \tag{14}$$

where σ is the growth rate of instability or rather the oscillation frequency if σ(= i ω with i = $(-1)^{½}$ the imaginary factor) is imaginary.

Consider an axisymmetric sinusoidal propagating wave along the gas-liquid interface. For a single Fourier term and based on the linearized perturbation technique, the perturbed radial distance of the gas cylinder is being

$$r = R_0 + \varepsilon_0 R_1 \tag{15}$$

with

$$R_1 = \exp(ikz + \sigma t) \tag{16}$$

The second term in the right side of equation (2.15) represents the elevation of the surface wave measured from the unperturbed position with k is the longitudinal wave number.

Based on the foregoing expansions, the relevant perturbation equations are given by:

$$\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0 \cdot \nabla)\underline{u}_1 - \frac{\mu}{\rho_0}(\underline{H}_0 \cdot \nabla)\underline{H}_1 = -\nabla \Pi_1 \tag{17}$$

$$\sigma \underline{H}_1 = (\underline{H}_0 \cdot \nabla)\underline{u}_1 - (\underline{u}_1 \cdot \nabla)\underline{H}_0 - (\underline{u}_0 \cdot \nabla)\underline{H}_1 - \underline{H}_0(\nabla \cdot \underline{u}_1) + \underline{u}_1(\nabla \cdot \underline{H}_0) \tag{18}$$

$$\nabla \cdot \underline{H}_1 = 0 \tag{19}$$

$$P_1 = a^2 \rho_1 \tag{20}$$

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \underline{u}_1 + \rho_1 \underline{u}_0) = 0 \tag{21}$$

$$\nabla \cdot \underline{H}_1^{gas} = 0 \tag{22}$$

$$\nabla \wedge \underline{H}_1^{gas} = 0 \tag{23}$$

and

$$P_{1s} = \frac{S}{R_0^2}\left(R_1 + R_0^2 \frac{\partial^2 R_1}{\partial z^2}\right) \tag{24}$$

where

$$\rho_0 \Pi_1 = P_1 + \frac{\mu}{2}(\underline{H} \cdot \underline{H})_1 \tag{25}$$

is the total magnetohydrodynamic pressure which is the sum of the perturbed kinetic pressure P_1 of the liquid and the magnetodynamic pressure (μ/2) $(\underline{H}.\underline{H})_1$, due to electromagnetic acting force. While a is the speed of sound in the compressible liquid defined by:

$$a = (\gamma P_0 / \; \rho_0)^{\frac{1}{2}} \tag{26}$$

By combining equations (2.20) and (2.21), we get

$$(\sigma + ikU)P_1 = -\rho_0 a^2 (\nabla \cdot \underline{u}_1)$$

In view of the time-space dependence and according to the linear perturbation technique used for solving the stability problems of cylindrical models (cf. Chandrasekhar (1980)and Radwan (2005)), every fluctuating quantity χ_1 (r,0,z, t) in the axisymmetric perturbation could be expressed as

$$\chi_1(r,\varphi,z,t) = \varepsilon_0 \chi_1^*(r)\exp(\sigma t + ikz) \qquad (27)$$

By the use of the expansion (27), the perturbed equations (17)-(24) are solved and the perturbed quantities $\underline{u}_1$, P_1, ρ_1, $\underline{H}_1$, $\underline{H}_1^g$, T are identified. These variables contain constants due to integration. Such constants may be determined upon applying appropriate boundary conditions.

Under the present circumstances, these boundary conditions are given as follows.

(i) The normal component u_r of the velocity vector $\underline{u}$ must be compatible with the velocity of the perturbed gas-liquid boundary across the interface (15) at r = R_0.

This condition yields

$$u_{1r} = \frac{\partial R_1}{\partial t} + (\underline{u}_0 \cdot \nabla)R_1 \qquad (28)$$

(ii) The jump of the normal component of the magnetic field vanishes across the liquid-gas interface at r = R_0.

This condition reads

$$\underline{N} . < \underline{H} > = 0 \qquad (29)$$

Up to first order, the condition (29) gives

$$\underline{N}_0 . < \underline{H}_1 > + \underline{N}_1 . < \underline{H}_0 > = 0 \qquad (30)$$

with

$$\langle \underline{H} \rangle = \underline{H}^{gas} - \underline{H}^{liquid} \qquad (31)$$

$$\underline{N}_0 = (1,0,0) \qquad (32)$$

$$\underline{N}_1 = (0,0,-ik)\exp(\sigma t + ikz) \qquad (33)$$

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(iii) The balance of the normal component of the total stress tensor across the gas-liquid interface at (r = R_o) is being

$$\rho_o \Pi_1 + R_1 \frac{\partial P_0}{\partial r} + \frac{\mu}{2} R_1 \frac{\partial(\underline{H}_0 \cdot \underline{H}_0)}{\partial r} = P_{1s} + \frac{\mu}{2}\left[\left(\underline{H}^{gas} \cdot \underline{H}^{gas}\right)_1 + R_1 \frac{\partial\left(\underline{H}_0^{gas} \cdot \underline{H}_0^{gas}\right)}{\partial r} \right] \qquad (34)$$

Consequently, after lengthy calculations, we obtain the following.

The total MHD pressure

$$\Pi_1 = -\frac{1}{\eta K_0'(y)}\left((\sigma + ikU)^2 + \Omega_A^2\right)K_0(\eta r)R_1 \qquad (35)$$

The magnetic field in the liquid region

$$\underline{H}_1 = \frac{ikH_0}{(\sigma + ikU)}\underline{u}_1 + \frac{H_0}{\rho_0 a^2}P_1 \underline{e}_2 \qquad (36)$$

The velocity components of the liquid

$$u_{1r} = -\frac{(\sigma + ikU)}{\left((\sigma + ikU)^2 + \Omega_A^2\right)}\frac{\partial \Pi_1}{\partial r} \qquad (37)$$

$$u_{1\phi} = 0 \qquad (38)$$

$$u_{1z} = ik(\sigma + ikU)\left[-1 + \frac{\mu H_0^2}{\xi a^2}\right]\left[(\sigma + ikU)^2 + \mu k^2 H_0^2\right]^{-1} \qquad (39)$$

The magnetic field in the gas region

$$\underline{H}_1^{gas} = \frac{i\alpha H_0}{I_0'(x)}\nabla\left(I_0(kr)R_1\right) \qquad (40)$$

The curvature pressure along the gas-liquid interface

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$$P_{1s} = \frac{S}{R_0^2}\left(1 - x^2\right)R_1 \quad (41)$$

with

$$\eta^2 = k^2 + \frac{(\sigma + ikU)^2}{a^2 \xi} \quad (42)$$

$$\xi = 1 + \frac{\mu H_0^2}{\rho(\sigma + ikU)^2}\left(\frac{(\sigma + ikU)^2}{a^2} + k^2\right) \quad (43)$$

Here x (= k R_o) is the ordinary longitudinal dimensionless wave number, y(= η R_o) the compressible longitudinal dimensionless wave number (where $\eta \to k$ as $a \to \infty$), I_0 and K_0 are the modified Bessel functions of the first and second kind of order zero, and $\Omega_A = \left(\mu H_0^2 k^2 / \rho_0\right)^{1/2}$ is Alfven wave frequency defined in terms of H_0.

By resorting to the foregoing solutions (12) and (35)–(43) of the basic equations in the unperturbed and perturbed states for compressibility condition (2.34), the following stability criterion is obtained

$$(\sigma + ikU)^2 = \frac{\mu H_0^2}{\rho_0 R_0^2}\left[-x^2 + \alpha^2 xy \frac{I_0(x) K_0'(y)}{I_0'(x) K_0(y)}\right] - \frac{S}{\rho_0 R_0^3}\left(1 - x^2\right)\left[\frac{y K_0'(y)}{K_0(y)}\right] \quad (44)$$

6 - Discussion and Limiting Cases

The dispersion relation (44) is valid for discussing the MHD stability of compressible hollow jet endowed with surface tension and acted by inertia and electromagnetic forces. This relation related the growth rate σ with the wave numbers x and y; the modified Bessel functions I_0 and K_0 of the first and second kinds of order zero and their derivatives, the parameters ρ_0, R_0, H_0, μ and S of\the problem and with the fundamental quantities $\left(\rho R_o^2 / \mu H_o^2\right)^{\frac{1}{2}}$ and $\left(\rho_o R_o^3 / S\right)^{\frac{1}{2}}$ as a unit of time.

The relation (44) is a general relation from which we may recover other reported works as limiting cases.

For an ideal hollow jet endowed with surface tension ($H_o = 0$ and $a \to \infty$) at rest initially (U=0), we have

$$\sigma^2 = \frac{-S}{\rho_o R_o^3}\left(1 - x^2\right)\frac{x K_1(x)}{K_o'(x)}, \qquad K_0'(x) = -K_1(x) \quad (45)$$

This relation has been given by Chandrasekhar (1981) for axisymmetric perturbation

If we assume that the fluid is incompressible ($a \to \infty$) and initially the fluid is at rest (U = 0), the dispersion relation (44), yields

$$\sigma^2 = \frac{S}{\rho_o R_o^3}\left(1 - x^2\right)\left[\frac{x K_1(x)}{K_o(x)}\right] + \frac{\mu H_o^2}{\rho_o R_o^2}\left[-x^2 - \alpha^2 x^2 \frac{I_o(x) K_1(x)}{I_1(x) K_o(x)}\right] \quad (46)$$

This is the magnetohydrodynamic dispersion relation of a hollow jet subjected by the capillary and MHD forces derived and documented by Radwan (1994).

The magnetodynamic dispersion relation of a streaming compressible hollow jet may obtained from equation (44), by just supposing (S = 0), in the form

$$(\sigma + ikU)^2 = \frac{\mu H_o^2}{\rho_o R_o^2}\left[-x^2 + \alpha^2 xy \frac{I_o(x) K_o'(y)}{I_o'(x) K_o(y)}\right] \quad (47)$$

The dispersion relation of a streaming compressible hollow jet subjected by the capillary force could be obtained from (44) as (H_o=0), in the form

$$(\sigma + ikU)^2 = -\frac{S}{\rho_o R_o^3}\left(1 - x^2\right)\left[\frac{y K_o'(y)}{K_o(y)}\right] \quad (48)$$

which is valid for all short and long wavelength.

7- Discussion and Results

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In order to investigate the instability and oscillation of the present model we have to write down about the characteristic and behaviour of the modified Bessel functions.
The recurrence relations of the modified Bessel functions(cf. Abramowitz and Stegun (1970)) are given by

$$2F'_m(x) = F_{m-1}(x) + F_{m+1}(x) \qquad (49)$$

where $F'_m(x)$ stands for $I'_m(x)$ and $-K'_m(x)$ while $F_m(x)$ stands for $I_m(x)$ and $K_m(x)$. By the use of relations (2.49) and the fact that $I_m(x)$ is positive definite and monotonic increasing while $K_m(x)$ is monotonic decreasing but never negative for non-zero real value of x, we have

$$I_0(x) > 0,\ K_0(x) > 0,\ x \neq 0 \qquad (50)$$

$$I'_0(x) > 0, K'_0(x) < 0 \qquad (51)$$

Based on the inequalities (2.50) and (51), we get

$$\frac{xK'_0(x)}{K_0(x)} < 0 \qquad (52)$$

$$\frac{x^2 I_0(x) K'_0(x)}{I'_0(x) K_0(x)} < 0 \qquad (53)$$

By utilizing (2.52) for (48) as (U = 0), we see that

$$\frac{\sigma^2}{\left(S/\rho_o R_o^3\right)^{1/2}} \leq 0, \qquad \text{as } 1 \leq x < \infty \qquad (54)$$

$$\frac{\sigma^2}{\left(S/\rho_o R_o^3\right)^{1/2}} > 0, \qquad \text{as } 0 < x < 1 \qquad (55)$$

This means that the cylindrical hollow jet is capillary unstable only for small domain of wave number while it is stable in all other domains.
From the view point of the inequality (53) the dispersion relation (47) reveals that both the magnetic fields pervaded in the gas and liquid regions have stabilizing effects. The stabilizing effect of the magnetic field in the gas region is valid for all short and long wavelengths. The analytical discussions indicate that the streaming has strong destabilizing effect.
Here we seek very important task concerning the effect of the compressibility on the stability of the hollow jet model which is in hand .
In the earlier studies of incompressible hollow jet by several authors (Chandrasekhar (1981), Drazin & Reid (1980), Cheng (1985), Kendall (1986), Radwan (1991)…. etc.) that give rise to the classical dispersion relation presuppose that the fluid moves incompressible i.e., that the divergence of the fluid velocity vanishes. that the compressibility has a stabilizing tendency. See also Chen (2003) and Shkadov& Sisoev (1996).

In reality the compressibility effects need careful treatment in each case of different models. Here we found that the incompressible fluid results are obtained as $a \to \infty$ (a is sound speed in the fluid). However for finite values of a (i.e. the fluid is compressible) it is expected that the growth rate values are larger than in the case of incompressible fluid. The unstable region of a compressible fluid is much larger than that of an incompressible fluid in the wave number domain of instability. This shows that, in our case of a hollow jet that the compressibility has a strong destabilizing tendency for all (short and long) wavelengths.
Any how such discussion and results could be judged and identified via the numerical analysis of the general dispersion relation (2.44) for different values of the different factors of the problem.

8 - Numerical Analysis

The dispersion relation (44) has been discussed numerically for all short and long wavelengths in which the dimensionless wave number is taken to be
$0 < x \leq 3$ and the corresponding values of σ or ω in the normal unit $\sqrt{\left(S/\ \rho R_0^3\right)}$ where (ω/2π is the frequency of oscillation) are determined. This has been performed for various values of $\left(H_0/\ H_s\right)$ and α. Then for every couple values of ($\left(H_0/\ H_s\right)$,α), different values of a is considered where $H_s = \sqrt{\left(S/\ \mu R_0\right)}$.

The numerical data are collected in tables, see tables (1) — (5) and presented in graphs, see figures (1) — (5). There are many features of interest in these tables and figures.

Corresponding to $((H_0/\ H_s),\alpha)$ = (0, 0.1) as a = 1,5,10,20 and 30; it is found that the unstable domains are 0 < x < 1.36928 , 0 < x < 1.133103 , 0 < x < 1.085419 , 0 < x < 1.050069 and 0 < x < 1.04138 , while the neighboring stable domains are given by 1.36928 < x < ∞ , 1.133103 < x < ∞ , 1.085419< x < ∞ , 1.050069 < x < ∞ and 1.04138 < x < ∞ . The critical points at which the transition from stable states to those of instability are occurred at x_c= 1.36928 , 1.133103 , 1.085419 , 1.050069 and 1.04138 respectively. See figure (1) and table (1) .

Corresponding to $((H_0/\ H_s),\alpha)$ = (0.1, 1) as a = 1,5,10,20 and 30; it is found that the unstable domains are 0 < x < 1.353 , 0 < x < 1.12614 , 0 < x < 1.07631 , 0 < x < 1.04187 and 0 < x < 1.03322 , while the neighboring stable domains are given by 1.353 < x < ∞ , 1.12614 < x < ∞ , 1.07631< x < ∞ , 1.04187 < x < ∞ and 1.03322 < x < ∞ . The critical points at which the transition from stable states to those of instability are occurred at x_c= 1.353 , 1.12614 , 1.07631 , 1.04187 and 1.03322 respectively. See figure (2) and table (2).

Corresponding to $((H_0/\ H_s),\alpha)$ = (0.3, 1) as a = 1,5,10,20 and 30; it is found that the model at hand is completely stable for all values of a for all short and long wavelengths. This means that the stabilizing effect of the magnetic field is predominating the compressibility destabilizing influence, and there is no any unstable state any more. See figure (3) and table (3).

Corresponding to $((H_0/\ H_s),\alpha)$ = (0.1, 2) as a= 1,5,10,20 and 30; it is found that the unstable domains are given by 0 < x < 1.84733 , 0 < x < 1.334 , 0 < x < 1.272 , 0 < x < 1.149 and 0 < x < 1.1036 , while the neighboring stable domains are given by 1.84733 < x < ∞ , 1.334 < x < ∞ , 1.272< x < ∞ , 1.149 < x < ∞ and 1.1036 < x < ∞ . The critical points at which the transition from stable states to those of instability are occurred at x_c= 1.84733 , 1.334 , 1.272 , 1.149 and 1.1036 respectively. See figure (4) and table (4).

Corresponding to $((H_0/\ H_s),\alpha)$ = (0.1, 3) as a = 1,5,10,20 and 30; it is found tha the unstable domains are 0 < x < 2.6997 , 0 < x < 1.75392 , 0 < x < 1.51354 , 0 < x < 1.336269 and 0 < x < 1.28833 , while the neighboring stable domains are given by 2.6997 < x < ∞ , 1.75392 < x <∞ , 1.51354 < x < ∞ , 1.336269 < x < ∞ and 1.28833 < x < ∞ . The critical points at which the transition from stable states to those of instability are occurred at x_c= 2.6997, 1.72392 , 1.51354 , 1.336269 and 1.28833 respectively. See figure (5) and table (5).

From the foregoing discussion we may conclude the following results.
1- The unstable domains are decreasing with increasing the values of compressibility parameter a. This means that the analytic results show that the compressibility is stabilizing and verified numerically.
2- The magnetic field parameter α is stabilizing.
3- The magnetic field is strong stabilizing whatever its smallest value.
4- The capillary force destabilizing effect may be suppressed by the stabilizing effect of the magnetic field and compressibility, and moreover stability exists.

II.9 Conclusions

The hydromagnetic instability of compressible hollow jet involved with surface tension is discussed in the axisymmetric mode for all short and long wavelengths. The dispersion relation is derived and discussed analytically and numerically. The axial magnetic fields inside the gas and liquid regions have stabilizing effects for all short and long wavelengths. This is physically interpreted that the axial field exerts a strong effect which causes the bending and twisting of the magnetic lines of force. The compressibility effects need careful treating. Here the incompressible fluid result is obtained as a tends to ∞ (a is the sound speed in the fluid). For finite value of a (i.e. compressible fluid), the temporal amplification is larger than that in the incompressible case. So the compressibility has a strong destabilizing tendency and increase the unstable domains. The streaming is destabilizing for all short and long wavelengths. The capillary force is destabilizing for small wave numbers while it is stabilizing for all the rest wavelengths. Whatever the stabilizing effect of the electromagnetic force is strong enough, the capillary, streaming and compressible instability could not be suppressed and the model will be always unstabl

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a / x	1	5	10	20	30
	σ*				
0.1	0.044306	0.066265	0.084976	0.126554	0.201214
0.2	0.087892	0.13092	0.167511	0.247905	0.384195
0.3	0.13	0.192263	0.245102	0.358985	0.535677
0.4	0.169735	0.248495	0.315062	0.454929	0.649015
0.5	0.206785	0.297595	0.37444	0.53099	0.722378
0.6	0.239604	0.337165	0.41975	0.582271	0.755831
0.7	0.267301	0.364184	0.446598	0.602968	0.748438
0.8	0.289524	0.3745	0.448642	0.584748	0.695334
0.9	0.301529	0.361497	0.415143	0.511877	0.581421
1	0.303891	0.312261	0.320587	0.336659	0.347728
				ω*	
1.1	0.291853	0.183633	0.13245	0.336192	0.413774
		ω*			
1.2	0.258438	0.261044	0.445926	0.651673	0.736311
1.3	0.185329	0.464747	0.671841	0.910042	0.998151
	ω*				
1.4	0.12339	0.646598	0.886071	1.152957	1.240371
1.5	0.291952	0.826442	1.099945	1.390446	1.474449
1.6	0.423332	1.009733	1.317232	1.626416	1.705356
1.7	0.548571	1.198649	1.539425	1.862726	1.935691
1.8	0.674062	1.394177	1.767119	2.10039	2.166979
1.9	0.802303	1.596778	2.000487	2.340015	2.400175
2	0.937195	1.806632	2.2395	2.581993	2.635893
2.1	1.071415	2.023769	2.484017	2.826595	2.874559
2.2	1.213371	2.248121	2.733847	3.074022	3.116456
2.3	1.360654	2.479573	2.988774	3.324425	3.36177
2.4	1.513433	2.717966	3.248584	3.577932	3.610665
2.5	1.671816	2.963123	3.513047	3.834619	3.863211
2.6	1.835879	3.214856	3.781984	4.094582	4.119466
2.7	2.005659	3.472967	4.055194	4.357866	4.379475
2.8	2.181185	3.737245	4.332505	4.624532	4.643242
2.9	2.362473	4.007493	4.613762	4.89463	4.910774
3	2.54952	4.283527	4.898826	5.168162	5.182075
x_c	1.36928	1.133103	1.085419	1.050069	1.04138

Table (1)
Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ Hs = 0.0, α = 0.1.

a / x	1	5	10	20	30
	σ*				
0.1	0.044045	0.065803	0.084321	0.125539	0.199549
0.2	0.087464	0.13	0.166259	0.245892	0.380959
0.3	0.129383	0.190893	0.243175	0.355921	0.530952
0.4	0.169086	0.246617	0.31241	0.450766	0.642853
0.5	0.205694	0.295161	0.370985	0.525623	0.714794
0.6	0.238265	0.334081	0.415367	0.575517	0.746693
0.7	0.265669	0.360319	0.441044	0.594508	0.737435
0.8	0.286557	0.369594	0.44152	0.573925	0.681689
0.9	0.299149	0.355092	0.405512	0.496991	0.563028
1	0.300965	0.303101	0.30527	0.309564	0.312615
				ω*	
1.1	0.288167	0.164165	0.170068	0.364678	0.443182
		ω*			
1.2	0.253476	0.275928	0.460576	0.66864	0.75452
1.3	0.177116	0.474647	0.683096	0.923553	1.012591
	ω*				
1.4	0.136638	0.654813	0.895779	1.164646	1.252749
1.5	0.298647	0.833775	1.108738	1.400968	1.485486
1.6	0.428602	1.016504	1.325398	1.63609	1.715401
1.7	0.553173	1.205027	1.547123	1.871751	1.945024
1.8	0.678233	1.400264	1.774455	2.108886	2.175721
1.9	0.806226	1.602623	2.007528	2.348084	2.408429
2	0.938243	1.812308	2.246293	2.589693	2.643738
2.1	1.074988	2.029286	2.490594	2.833976	2.882053
2.2	1.21684	2.253513	2.740234	3.081087	3.123644
2.3	1.364001	2.484848	2.994991	3.331276	3.368694
2.4	1.516707	2.723142	3.254643	3.58455	3.617347
2.5	1.675052	2.968208	3.518977	3.841042	3.869625
2.6	1.839049	3.21986	3.787783	4.100817	4.125736
2.7	2.008781	3.477887	4.060874	4.363932	4.385567
2.8	2.184262	3.742098	4.338064	4.630443	4.649172
2.9	2.365523	4.012281	4.619221	4.900388	4.916554
3	2.552548	4.288251	4.904182	5.17379	5.187991
x_c	1.35300	1.12614	1.07631	1.04187	1.03322

Table (2)
Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ Hs = 0.1, α = 1.

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a / x	1	5	10	20	30
			ω*		
0.1	0.302704	0.308794	0.315737	0.336419	0.388201
0.2	0.605302	0.617237	0.630793	0.670559	0.763643
0.3	0.907695	0.925095	0.944495	1.00035	1.118168
0.4	1.209773	1.231666	1.256185	1.324047	1.449655
0.5	1.51143	1.536945	1.565241	1.640366	1.760224
0.6	1.812567	1.840516	1.871096	1.948461	2.053801
0.7	2.113055	2.142032	2.1732	2.247986	2.334395
0.8	2.412851	2.441176	2.471141	2.538838	2.605218
0.9	2.711789	2.737634	2.764435	2.821294	2.868606
1	3.009794	3.031117	3.052769	3.095721	3.126148
1.1	3.306751	3.32134	3.335821	3.362618	3.378846
1.2	3.602555	3.608047	3.613364	3.622513	3.627272
1.3	3.897114	3.890977	3.88519	3.875926	3.87177
1.4	4.190322	4.16988	4.151181	4.1233	4.11253
1.5	4.482053	4.444547	4.411224	4.365066	4.349598
1.6	4.772232	4.714764	4.665276	4.601554	4.582979
1.7	5.060751	4.980341	4.913298	4.833084	4.81265
1.8	5.347495	5.241088	5.155298	5.059852	5.038561
1.9	5.632362	5.496845	5.391299	5.282045	5.260637
2	5.91526	5.74746	5.621343	5.499791	5.478814
2.1	6.196079	5.992796	5.845477	5.713178	5.693022
2.2	6.47472	6.232728	6.063769	5.922255	5.903186
2.3	6.751074	6.46714	6.276281	6.127055	6.109223
2.4	7.025048	6.695924	6.483078	6.327582	6.311085
2.5	7.296533	6.918996	6.684235	6.523826	6.508679
2.6	7.565421	7.136267	6.879797	6.715743	6.70194
2.7	7.831628	7.347666	7.069844	6.903311	6.881012
2.8	8.095036	7.553119	7.254412	7.086459	7.075168
2.9	8.355543	7.752567	7.433546	7.265143	7.255033
3	8.61306	7.948792	7.607286	7.439281	7.430249

Table (3)
Values of the oscillation frequency ω* for Ho/ Hs =0.3 , α = 1.

a / x	1	5	10	20	30
			σ*		
0.1	0.059582	0.077253	0.093691	0.132337	0.204494
0.2	0.118701	0.153118	0.185176	0.259715	0.391139
0.3	0.176761	0.226168	0.272195	0.377333	0.547083
0.4	0.233165	0.294907	0.352392	0.480645	0.666018
0.5	0.287367	0.357729	0.423261	0.565367	0.746586
0.6	0.338748	0.412832	0.48198	0.627256	0.789487
0.7	0.386639	0.458138	0.525205	0.661695	0.794921
0.8	0.430325	0.49108	0.548726	0.662741	0.760422
0.9	0.468967	0.50834	0.546553	0.620878	0.677547
1	0.501627	0.505183	0.508793	0.515946	0.521018
1.1	0.527171	0.473878	0.414005	0.269141	0.110091
				ω*	
1.2	0.544197	0.398748	0.161648	0.450999	0.568168
			ω*		
1.3	0.550908	0.224967	0.432643	0.753571	0.858914
		ω*			
1.4	0.54494	0.329909	0.691788	1.013287	1.112425
1.5	0.522638	0.572678	0.92617	1.259087	1.35178
1.6	0.478059	0.78457	1.155076	1.499497	1.58514
1.7	0.399249	0.990252	1.384597	1.738102	1.816387
1.8	0.248435	1.196746	1.617062	1.976821	2.04765
	ω*				
1.9	0.262044	1.407018	1.853591	2.214	2.280221
2	0.485994	1.622433	2.0947	2.458233	2.514917
2.1	0.667428	1.843665	2.340528	2.702047	2.752283
2.2	0.837437	2.071111	2.591092	2.948391	2.992686
2.3	1.004356	2.304897	2.846317	3.197499	3.236371
2.4	1.171734	2.545036	3.106081	3.449536	3.483533
2.5	1.341413	2.791469	3.370237	3.704646	3.734274
2.6	1.514464	3.044142	3.63864	3.962928	3.988671
2.7	1.691567	3.302878	3.911138	4.224476	4.246775
2.8	1.873126	3.567548	4.187589	4.489343	4.508625
2.9	2.05949	3.837981	4.467852	4.757604	4.774233
3	2.250844	4.114013	4.751831	5.029274	5.04359
x_c	1.84733	1.334	1.272	1.149	1.10361

Table (4)
Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ Hs = 0.1, α = 2.

a / x	1	5	10	20	30
x			σ*		
0.1	0.086493	0.097468	0.11327	0.14757	0.216035
0.2	0.172624	0.198446	0.224718	0.290666	0.414934
0.3	0.258019	0.294968	0.332492	0.425006	0.584543
0.4	0.342301	0.388214	0.434718	0.546681	0.719354
0.5	0.4251	0.477032	0.529501	0.652173	0.818933
0.6	0.506034	0.560205	0.614809	0.738336	0.885167
0.7	0.584671	0.636428	0.688518	0.802229	0.92012
0.8	0.660621	0.704294	0.748238	0.840797	0.924651
0.9	0.733417	0.762194	0.791221	0.850347	0.897437
1	0.802608	0.808295	0.81407	0.825524	0.833607
1.1	0.867692	0.840375	0.81225	0.757087	0.721412
1.2	0.928127	0.855652	0.779038	0.624944	0.527257
					ω*
1.3	0.983326	0.850382	0.702816	0.356403	0.191565
				ω*	
1.4	1.032642	0.819151	0.55752	0.47244	0.654599
1.5	1.075346	0.753072	0.218518	0.815714	0.950053
			ω*		
1.6	1.110603	0.63456	0.552024	1.097497	1.210174
1.7	1.137462	0.410663	0.869299	1.358418	1.455974
		ω*			
1.8	1.154786	0.379579	1.146194	1.61013	1.69542
1.9	1.161189	0.738072	1.409677	1.857671	1.932403
2	1.154946	1.021088	1.668463	2.10364	2.169097
2.1	1.133821	1.283328	1.926372	2.34964	2.406829
2.2	1.094751	1.538467	2.185328	2.596594	2.646466
2.3	1.033276	1.792097	2.446361	2.845177	2.88856
2.4	0.937497	2.046981	2.71005	3.09586	3.133508
2.5	0.808554	2.30463	2.976715	3.348985	3.381582
2.6	0.599667	2.565913	3.246521	3.604802	3.63296
	ω*				
2.7	0.01005	2.831334	3.519574	3.863496	3.887789
2.8	0.656917	3.10118	3.795866	4.125239	4.146143
2.9	0.969897	3.375604	4.075426	4.390137	4.408095
3	1.238556	3.654668	4.352126	4.658272	4.673682
x_c	2.6997	1.75392	1.51354	1.336269	1.28833

Table (5)
Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ Hs = 0.1, α = 3.

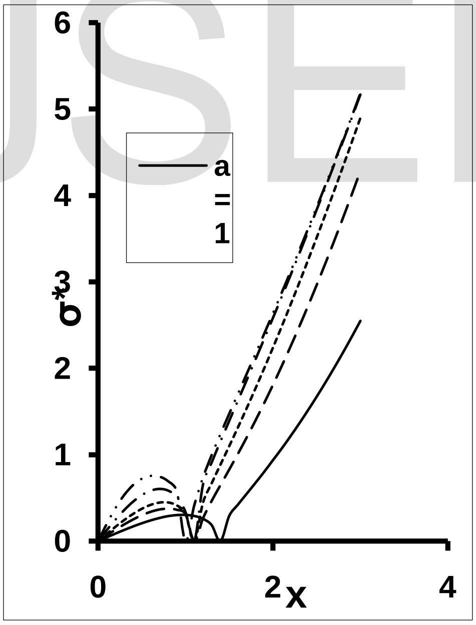

Figure (1) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for $H_0/H_s = 0.1$, α =1

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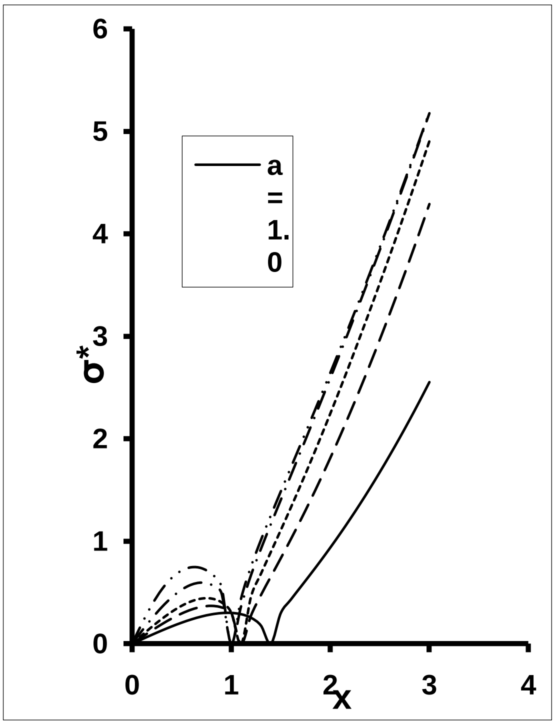

Figure (2) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for $H_0/H_s=$ 0.1, α =2, U^*

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Figure (3) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/H_s= 0.3, 0.1, α =1, U^*=0.

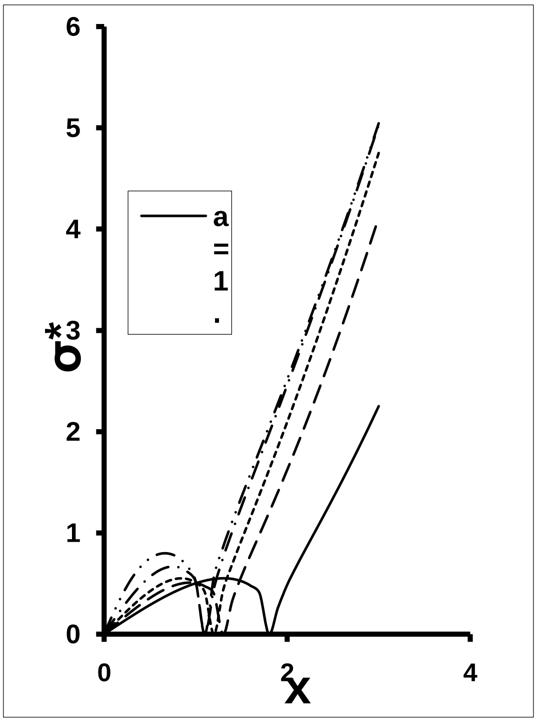

Figure (4) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/H_s= 0.1, α =2, U^*=0.

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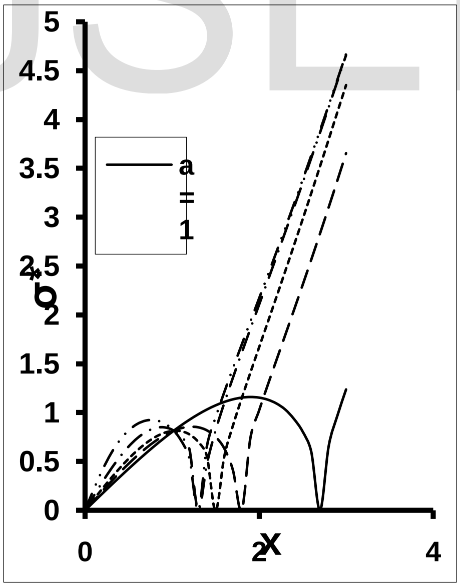

Figure (5) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/H_s= 0.1, α =3, U^*=0.

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